

# RR - Dilaton Interaction In a Type IIB Superstring

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## Abstract

We analyze the interaction between the massless Ramond-Ramond states with a dilaton in a type II superstring. By constructing vertex operators for massless Ramond-Ramond states and computing their correlation functions with a dilaton we find the Ramond-Ramond part of the superstring low-energy effective action. Those RR terms appear in the action without the standard dilatonic factor (string coupling constant) as has been shown earlier on the basis of space-time supersymmetry. The geometrical interpretation of this fact is presented. Namely we argue that the spin operators in the RR vertices effectively decrease the Euler character of the worldsheet by 1 unit. As a result, the dilaton term in the worldsheet action has the form:  $\sim \int d^2z e^\Phi \Lambda F \bar{\Lambda}$ , where  $\Lambda$  and  $\bar{\Lambda}$  are the  $(1,0)$  and  $(0,1)$  spin operators for the ghost and matter fields,  $F$  is the RR field strength contracted with  $\gamma$ -matrices. We also analyse the interaction of a dilaton with the massive RR states.

October 95

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Recently there has been a significant progress in the understanding of string-string dualities. The important step in analyzing the dualities is to study low-energy effective actions of different string theories. To obtain the low-energy Lagrangian for a given string theory, one has to compute the couplings between its massless states. In this paper, we analyze how Ramond-Ramond states interact with a dilaton in a closed superstring and derive RR contributions to the effective Lagrangian directly from superstring theory. It is of interest to present string-theoretical derivation of these couplings and to generalise it to massive states. Let us explain the idea of the computation. The bosonic part of the worldsheet superstring action perturbed with background fields is given by:

$$I = \frac{1}{2\pi\alpha} \int d^2z \frac{1}{2} \sqrt{\gamma} \gamma^{ab} g_{ij}(X) + \epsilon^{ab} B_{ij} \partial_a X^i \partial_b X^j + \frac{1}{4\pi} \int d^2z \sqrt{\gamma} R^{(2)} \Phi(X) + V_i T_i(X) \quad (1)$$

Here  $B_{ij}$  is an axion,  $\Phi$  is the space-time field of a dilaton,  $T_i$  are background space-time fields and  $V_i$  are the corresponding vertex operators. The corresponding low - energy Lagrangian consists of the standard NS-NS part: [1]

$$L_{NS-NS} \sim \int \sqrt{g} e^{-2\Phi} [-R + 4\nabla\Phi\nabla\Phi + \frac{1}{12} H_{ijk} H^{ijk}], \quad (2)$$

where  $H_{ijk} = 3\partial_{[i} B_{jk]}$ , and the RR part which is the sum over the contributions from the antisymmetric field strengths:

$$L_{RR} = \sum_j \int \sqrt{g} g^{\mu_1\mu_1'} \dots g^{\mu_j\mu_j'} F_{\mu_1\dots\mu_j} F_{\mu_1'\dots\mu_j'} \quad (3)$$

where  $F_{\mu_1\dots\mu_j}$  are closed  $j$ -forms and the summation goes over even  $j$  for the type IIA and odd  $j$  for the type IIB superstrings. The well known peculiar feature of the RR sector is the absence of the factor  $e^{-2\Phi}$  in front of the effective action [2,3]. It is usually derived from the 10d supersymmetry [4]. However, from the string-theoretical point of view this result is strange since the factor of  $e^{-2\Phi}$  is thought to be universal, coming from the fact that the Euler character of the sphere is equal to -2. Indeed, under the translation  $\Phi \rightarrow \Phi + C$  (where  $C$  is constant) the expression (1) transforms as  $I \rightarrow I + C\chi = I - 2C$ ;  $\chi$  is the Euler character for our world surface. It follows therefore that the effective action is multiplied by  $e^{C\chi} = e^{-2C}$  under this transformation. In this paper we shall try to explain this form of the effective action (3) from the string-theoretical point of view. In order to obtain effective action from string theory we have to compare 3-point functions coming from the computation in string theory with those from the low-energy Lagrangian. To do

that we have to recast the Lagrangian in the form in which the dilaton is not mixing with gravity. The necessary field redefinition is well-known and is given by:  $g^{\mu\nu} \rightarrow e^{\frac{-4\Phi}{D-2}} g^{\mu\nu}$  after which the action takes the form:

$$\begin{aligned} L_{NS-NS} &\sim \int d^{10}X \sqrt{g} (-R + 4\nabla\Phi\nabla\Phi + \frac{1}{12} e^{\frac{8\Phi}{D-2}} H_{ijk} H^{ijk}) \\ L_{RR} &\sim \int d^{10}X \sqrt{g} (-\frac{1}{4} e^{\frac{3\Phi}{2}} F_{\mu\nu} F^{\mu\nu} - \frac{1}{32} e^{\frac{\Phi}{2}} S_{\alpha\beta\gamma\delta} S^{\alpha\beta\gamma\delta}). \end{aligned} \quad (4)$$

Here  $S_{\alpha\beta\gamma\delta} = 4\partial_{[\alpha} S_{\beta\gamma\delta]}$ , and the 3-form  $C_{\beta\gamma\delta}$  and the vector potential  $A_\mu$  constitute the Ramond-Ramond massless spectrum in a type IIB superstring. All we have to do now is to extract the  $\Phi FF$  and  $\Phi SS$  couplings from string theory.

Note that it is also possible however to recast this action into the standard string theory form by redefining the fields. By using the redefinition:  $A_\mu \rightarrow e^{-\Phi} B_\mu$ ,  $C_{\beta\gamma\delta} \rightarrow e^{-\Phi} D_{\beta\gamma\delta}$  we may write the action in the form

$$S^{RR} \sim \int d^{10}X \sqrt{g} e^{-2\Phi} (-\frac{1}{4} (\partial_{[\mu} B_{\nu]} - \partial_{[\mu} \Phi B_{\nu]})^2 - \frac{1}{32} (\partial_{[\alpha} D_{\beta\gamma\delta]} - \partial_{[\alpha} \Phi D_{\beta\gamma\delta]})^2 + \dots) \quad (5)$$

Though the factor  $e^{-2\Phi}$  does appear in the action written in the form (4), the gauge terms have now a very non-standard form. Now, the string coupling  $g_{st}$  enters just as in the NS-NS sector. But the price we pay for that are the extra cubic and quartic interactions between the dilaton and gauge fields which is seen from (5). Of course the elements of  $S$ -matrix are invariant under field redefinition and are the same for (4) and (5). It is more convenient to compare 3-point functions from string theory with those following from the Lagrangian (4) containing unrescaled fields. So, if we want to check that string theory really gives (4), we must compute the 3-point functions involving dilaton and two RR states and show that their value is equal to the one following from (4). This is what we will do below and then give a geometrical interpretation of the result. As we have mentioned above, the RR sector in the type II superstring theory has the following massless excitations:  $A_\mu$  - the vector; and  $C_{\alpha\beta\gamma}$  - the 3-form. The corresponding vertex operators must have the form [5]

$$\begin{aligned} V_1(k) &= \frac{1}{2} F_{\mu\nu}(k) [\gamma^\mu, \gamma^\nu]_{AB} \Sigma_A \bar{\Sigma}_B e^{-1/2(\phi+\bar{\phi})} e^{ikX} \\ V_2(k) &= \frac{1}{24} S_{\alpha\beta\gamma\delta}(k) \gamma^{[\alpha} \gamma^\beta \gamma^\gamma \gamma^{\delta]} \Sigma \bar{\Sigma} e^{-1/2(\phi+\bar{\phi})} e^{ikX}. \end{aligned} \quad (6)$$

Here  $\Sigma, \bar{\Sigma}$  are the spin operators for matter fields, and  $\phi, \bar{\phi}$  are the bosonized superconformal ghosts. The requirement of BRST invariance imposes certain constraints on the

polarization tensors in  $V_1$  and  $V_2$ . To demonstrate this, let us apply the left BRST charge to the first operator in (6). The BRST invariance condition is

$$0 = \{Q_{BRST}, V_1(k)\} = e^{1/2\phi-\chi}\Sigma(\gamma k)\gamma^{[\mu}\gamma^{\nu]}F_{\mu\nu}\bar{\Sigma}e^{ikX} \quad (7)$$

Here  $\chi$  is the auxiliary field in the bosonization formulas for the superconformal ghosts  $\beta, \gamma: \gamma = e^{\phi-\chi}, \beta = e^{\chi-\phi}\partial\chi$ . The condition (7) requires therefore that

$$F_{\mu\nu}(k)\Sigma(\gamma k)\gamma^{[\mu}\gamma^{\nu]}\bar{\Sigma} = 0 \quad (8)$$

This is satisfied if:

$$\begin{aligned} (k_\alpha F_{\mu\nu} + k_\mu F_{\nu\alpha} + k_\nu F_{\alpha\mu}) &= 0, \\ k_\mu F_{\mu\nu} &= 0 \end{aligned} \quad (9)$$

-the Maxwell's equations. From these equations it follows that  $F_{\mu\nu}$  can be represented as  $F_{\mu\nu}(k) = 1/2(e_\mu(k)k_\nu - e_\nu(k)k_\mu)$  where the polarization vector satisfies  $(e(k)k) = 0$ . Analogously, the BRST condition for  $V_2$  fixes  $S_{\alpha\beta\gamma\delta}(k) = \frac{1}{24}k_{[\alpha}s_{\beta\gamma\delta]}(k)$  where  $s_{\beta\gamma\delta}$  is some antisymmetric tensor satisfying the transversality condition:  $k_\beta s_{\beta\gamma\delta} = 0$ . Therefore the BRST-invariant type IIB RR massless vertices are

$$\begin{aligned} e_\mu(k)V^\mu(k) &= e_\mu(k)e^{-1/2(\phi+\bar{\phi})}\Sigma(\gamma k)\gamma^\mu\bar{\Sigma}e^{ikX} \\ s_{\alpha\beta\gamma}(k)V^{\alpha\beta\gamma} &= 1/6s_{\alpha\beta\gamma}(k)(\gamma k)\gamma^{[\alpha}\gamma^\beta\gamma^{\gamma]}e^{ikX} \end{aligned} \quad (10)$$

. The equivalent formulas for the massless RR vertices have been proposed in [5] (see also [3]) Now that we have the vertices corresponding to the RR states, let us compute their couplings with the dilaton vertex  $V_\Phi$ . The vertex operator of the dilaton is given by [6]  $V_\Phi^{(0)} = e_{\mu\nu}(k)\partial X^\mu\bar{\partial}X^\nu e^{ikX}$ , where  $e_{\mu\nu}(k) = \eta_{\mu\nu} - k_\mu\bar{k}_\nu - \bar{k}_\mu k_\nu$ . Here  $\eta_{\mu\nu}$  is the flat space-time metric and  $\bar{k}$  is defined so that  $\bar{k}^2 = 0, (k\bar{k}) = 1$ . For our computations it will be more convenient to use the dilaton emission vertex in another picture:  $V_\Phi = e_{\mu\nu}(k)e^{-\phi-\bar{\phi}}\psi^\mu\bar{\psi}^\nu$ . The  $V_\Phi^{(0)}$  is obtained by the left and right picture-changings of  $V_\Phi$ . The correlation function of 2 massless vector gauge particles with a dilaton is

$$\begin{aligned} &e_\mu(k_1)e_\nu(k_2)(\eta_{\alpha\beta} - k_\alpha\bar{k}_\beta - \bar{k}_\alpha k_\beta) < V_\mu(k_1)V_\nu(k_2)e^{-\phi-\bar{\phi}}\psi^\alpha\bar{\psi}^\beta > = \\ &= < e^{-1/2(\phi+\bar{\phi})}\Sigma_A\Sigma_B\gamma^\mu(\gamma k_1)e^{ik_1X}(z_1, \bar{z}_1)e^{-1/2(\phi+\bar{\phi})}\Sigma_C\Sigma_D\gamma^\nu(\gamma k_2)e^{ik_2X}(z_2, \bar{z}_2) \times \\ &\quad \times e^{-\phi-\bar{\phi}}\psi_\alpha\bar{\psi}_\beta e^{ikX}(z_3, \bar{z}_3) > e^\mu(k_1)e^\nu(k_2)(\eta_{\alpha\beta} - k_\alpha\bar{k}_\beta - \bar{k}_\alpha k_\beta) = \quad (11) \\ &= \frac{1}{(z_1 - z_2)(z_2 - z_3)(z_3 - z_1)(\bar{z}_1 - \bar{z}_2)(\bar{z}_2 - \bar{z}_3)(\bar{z}_3 - \bar{z}_1)} e_\mu(k_1)e_\nu(k_2) \times \\ &\quad \times (\eta_{\alpha\beta} - k_\alpha\bar{k}_\beta - \bar{k}_\alpha k_\beta) Tr[(\gamma k_1)\gamma^\mu\gamma^\alpha(\gamma k_2)\gamma^\nu\gamma^\beta], \end{aligned}$$

with  $k = -k_1 - k_2$ . The contribution to the 3-point function is therefore

$$\begin{aligned}
C_{A_\mu A_\nu \Phi} = & 1/2 \text{Tr}[(\gamma k_1)(\gamma e(k_1))\gamma^\alpha(\gamma k_2)(\gamma e(k_2))\gamma_\alpha] - \\
& -1/2 \text{Tr}[(\gamma k_1)(\gamma e(k_1))(\gamma k)(\gamma k_2)(\gamma e(k_2))(\gamma \bar{k})] - \\
& -1/2 \text{Tr}[(\gamma k_1)(\gamma e(k_1))(\gamma \bar{k})(\gamma k_2)(\gamma e(k_2))(\gamma k)]
\end{aligned} \tag{12}$$

The first ,second and third terms in (7) contribute

$$\begin{aligned}
C_{A_\mu A_\nu \Phi}^{(1)} = & 1/2 \text{Tr}[(\gamma k_1)(\gamma e(k_1))\gamma^\alpha(\gamma k_2)(\gamma e(k_2))\gamma_\alpha] = \\
= & -1/2 \text{Tr}[(\gamma e(k_1))(\gamma k_2)(\gamma e(k_2))(\gamma k_1)] + 1/2 \text{Tr}[(\gamma e(k_1))(\gamma k_1)(\gamma k_2)(\gamma e(k_2))] - \\
& -1/2(k_1 e(k_2)) \text{Tr}[(\gamma e(k_1))\gamma^\alpha(\gamma k_2)\gamma_\alpha] = \\
= & -1/2((e(k_1)k_2)(e(k_2)k_1) - (e(k_1)k_2)(e(k_2)k_1) - \\
& - (e(k_2)k_1)[2e(k_1)k_2 - D(e(k_1)k_2)]) = 1/2(D-4)(k_1 e(k_2))(k_2 e(k_1)) \\
C_{A_\mu A_\nu \Phi}^{(2)} = & C_{A_\mu A_\nu \Phi}^{(3)} = -1/2 \text{Tr}[(\gamma k_1)(\gamma e(k_1))(\gamma k)(\gamma k_2)(\gamma e(k_2))(\gamma \bar{k})] = \\
= & 1/2(k_1 e(k_2)) \text{Tr}[(\gamma e(k_1))(\gamma k)(\gamma k_2)(\gamma \bar{k})] - 1/2(k_1 \bar{k}) \text{Tr}[(\gamma e(k_1))(\gamma k)(\gamma k_2)(\gamma e(k_2))] = \\
= & 1/2(k_1 e(k_2))[-(k_2 e(k_1))(k_2 \bar{k}) - (k_2 e(k_1))(k \bar{k})] - \\
& -1/2(k_1 \bar{k})(e(k_1)k_2)(e(k_2)k_1) = 0
\end{aligned} \tag{13}$$

therefore

$$C_{A_\mu A_\nu \Phi} = 3(k_1 e(k_2))(k_2 e(k_1)) \tag{14}$$

Note that for  $D = 4$  this 3-point function would be zero, as has been shown in [7] Hence the corresponding term in the effective Lagrangian will be  $\sim 3/8 F_{\mu\nu} F^{\mu\nu} \Phi(X)$ . Another 3-point function contributing to the interaction of a dilaton with the massless RR states is that of  $V_\Phi$  with 2 gauge 3-forms. By using the vertex operators (10) we write the correlation function as

$$\begin{aligned}
& s_{\alpha\beta\gamma}(k_1) s_{ijk}(k_2) < V^{\alpha\beta\gamma}(z_1, k_1) V^{ijk}(z_2, k_2) V_\Phi(z_3, k_3) > = \\
= & \frac{1}{36} s_{\alpha\beta\gamma}(k_1) s_{ijk}(k_2) [(\gamma k_1) \gamma^{[\alpha} \gamma^\beta \gamma^{\gamma]}]_{AB} [(\gamma k_2) \gamma^{[i} \gamma^j \gamma^{k]}]_{CD} (\eta_{\mu\nu} - k_\mu \bar{k}_\nu - \bar{k}_\mu k_\nu) \times \\
& \times < e^{-1/2(\phi+\bar{\phi})} \Sigma_A \bar{\Sigma}_B e^{ik_1 X}(z_1, \bar{z}_1) e^{-1/2(\phi+\bar{\phi})} \Sigma_C \bar{\Sigma}_D e^{ik_2 X}(z_2, \bar{z}_2) \times \\
& \times e^{-\phi-\bar{\phi}} \psi^\mu \bar{\psi}_\nu e^{ik X}(z_3, \bar{z}_3) >
\end{aligned} \tag{15}$$

The 3-point function is hence given by

$$\begin{aligned}
C_{cc\Phi} &= \frac{1}{36} e_{\mu\nu}(k) s_{\alpha\beta\gamma}(k_1) s_{ijk}(k_2) 1/2 \text{Tr}[(\gamma k_1) \gamma^{[\alpha} \gamma^\beta \gamma^\gamma] \gamma^\mu (\gamma k_2) \gamma^{i] \gamma^j \gamma^k] \gamma^\nu] = \\
&= \frac{1}{72} s_{\alpha\beta\gamma}(k_1) s_{ijk}(k_2) \text{Tr}[(\gamma k_1) \gamma^{[\alpha} \gamma^\beta \gamma^\gamma] \gamma^\mu (\gamma k_2) \gamma^{i] \gamma^j \gamma^k] \gamma_\mu] - \\
&\quad - \frac{1}{72} s_{\alpha\beta\gamma}(k_1) s_{ijk}(k_2) \text{Tr}[(\gamma k_1) \gamma^{[\alpha} \gamma^\beta \gamma^\gamma] (\gamma k) (\gamma k_2) \gamma^{i] \gamma^j \gamma^k] (\gamma \bar{k})] - \\
&\quad - \frac{1}{72} s_{\alpha\beta\gamma}(k_1) s_{ijk}(k_2) \text{Tr}[(\gamma k_1) \gamma^{[\alpha} \gamma^\beta \gamma^\gamma] (\gamma \bar{k}) (\gamma k_2) \gamma^{i] \gamma^k \gamma^l] (\gamma k)] = \\
&= \frac{1}{72} (C_{cc\Phi}^{(1)} + C_{cc\Phi}^{(2)} + C_{cc\Phi}^{(3)})
\end{aligned} \tag{16}$$

The 3 terms in the r.h.s. contribute:

$$\begin{aligned}
C_{cc\Phi}^{(1)} &= 72 k_{1\alpha} s_{\delta\beta\gamma}(k_1) k_{2\delta} s_{\alpha\beta\gamma}(k_2); \\
C_{cc\Phi}^{(2)} &= C_{cc\Phi}^{(3)} = 0;
\end{aligned} \tag{17}$$

The antisymmetrization over  $\alpha, \beta, \gamma, \delta$  is implied.

The 3-point function (15) is equal to

$$C_{cc\Phi} = (k_{1[\alpha} s_{\delta\beta\gamma]}(k_2)) (k_{2[\delta} s_{\alpha\beta\gamma]}(k_2)) \tag{18}$$

The corresponding contribution to the low-energy effective space-time Lagrangian then will be  $\sim \frac{1}{64} S_{\alpha\beta\gamma\delta} S^{\alpha\beta\gamma\delta} \Phi(X)$ , where  $S_{\alpha\beta\gamma\delta}(X) = 4 \partial_{[\alpha} C_{\beta\gamma\delta]}(X)$ .

Now that we know from (14),(18) the couplings of the massless RR states with a dilaton it is easy to write down the massless RR-terms interacting with a space-time dilaton in the low-energy effective action:

$$\begin{aligned}
S_{eff}^{RR\Phi} &\sim \int d^{10} X \sqrt{g} \left( -\frac{1}{4} e^{3/2\Phi} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} e^{1/2\Phi} (\partial_{[\alpha} C_{\beta\gamma\delta]})^2 \right) = \\
&= \int d^{10} X \sqrt{g} \left( -\frac{1}{4} e^{3/2\Phi} F_{\mu\nu} F^{\mu\nu} - \frac{1}{32} e^{1/2\Phi} S_{\alpha\beta\gamma\delta} S^{\alpha\beta\gamma\delta} \right)
\end{aligned} \tag{19}$$

We see therefore that the string perturbation theory gives the same effective Lagrangian as in (4), that followed from the 10d supersymmetry arguments. By making the inverse rescaling we may return to the effective action written in the form (3). Thus, we have shown that the string perturbation theory does give the low-energy limit with the factor  $e^{-2\Phi}$  absent in the Ramond-Ramond effective action.

Let us try to give the geometrical interpretation to these results. We will argue that the correct contribution of the RR states to the worldsheet action is given by:

$$I_{RR} \sim \int d^2 z e^{\Phi(X)} \Lambda(\gamma^{[\mu} \gamma^{\nu]} F_{\mu\nu} + \dots) \bar{\Lambda} \tag{20}$$

where  $\Lambda$  is a total (ghost + matter) spin operator [8]:  $\Lambda = \Sigma_{gh}\Sigma$  and  $F$  is RR field strength. This would give an extra factor  $e^{2\Phi}$  in the target space action and the formula (3). The origin of the  $e^\Phi$  factor can be interpreted as follows. When we define the RR state we have to cut a small hole in the surface and to require fermions to be antiperiodic around this hole. Cutting the hole reduces Euler character by 1 unit. Hence we conjecture that any insertion of the RR state must be accompanied by the factor  $e^\Phi$  in the worldsheet action. The absence of  $g_{st}$  in (3) is now simply explained by the fact that a sphere with 2 holes has zero Euler character. The formula (20) has two important consequences. First of all, we should expect that higher order corrections to the RR effective action should be multiplied by the various dilatonic factors  $\sim e^{(n-2)\Phi}$  where  $n$  is the number of insertions of the spin operators  $\Sigma$  (and  $\bar{\Sigma}$ ) in the correlation functions contributing to the corresponding higher order corrections to the  $\beta$ -function. Next, the terms corresponding to the coupling of massive RR states with the dilaton  $\Phi$  should also (in the lowest order) appear in the action without any dilatonic factor, exactly as in the case of the massless RR states considered above. Note that it is still possible to apply the string perturbation theory to the massive RR - dilaton interactions since the field of the dilaton is changing slowly. Let us check the proposal regarding the massive RR- dilaton couplings in the case of the massive RR states at the lowest level with  $k^2 = 2$  (the lowest massive level). Since in this case we have  $\dim(e^{ikX}) = -1$ , the most general gauge-invariant expression for the RR operator of dimension 1 with  $k^2 = 2$  is

$$V^{\mu_1 \dots \mu_{2n-1} \alpha \beta} = \frac{1}{(2n-1)!} e^{-1/2(\phi+\bar{\phi})} \Sigma \gamma^{[\mu_1} \dots (\gamma^{\mu_{2n-1}} \gamma^{\mu_{2n}}] k_{\mu_{2n}} - k^{\mu_{2n-1}}] \bar{\Sigma} \times \quad (21)$$

$$\times (\partial X^\alpha + iA(k\psi)\psi^\alpha)(\bar{\partial} X^\beta - iB(k\bar{\psi})\bar{\psi}^\beta) e^{ikX}$$

where  $A$  and  $B$  are some numbers. The condition of the BRST invariance fixes  $A = B = 1$ . For simplicity, let us consider the case of the emission of a massive (with  $k^2 = 2$ ) vector particle  $D_\mu(X)$  and compute the interaction of 2 gauge particles with the dilaton  $\Phi$ . It follows from (21) that the vertex operator of the emission of a massive ( $k^2 = 2$ ) vector boson is given by

$$V_D^\mu(k) = e^{-1/2(\phi+\bar{\phi})} \left[ \frac{1}{8} \Sigma_A (\gamma^\mu (\gamma k) - k^\mu)_{AB} \bar{\Sigma}_B (\partial X^\alpha + i(k\psi)\psi^\alpha) (\bar{\partial} X_\alpha - i(k\bar{\psi})\bar{\psi}_\alpha) e^{ikX} + \right.$$

$$+ \Sigma((\gamma k)\gamma^\alpha - k^\alpha) \bar{\Sigma}(\partial X_\alpha + i(k\psi)\psi_\alpha) (\bar{\partial} X^\mu - i(k\bar{\psi})\bar{\psi}^\mu) e^{ikX} +$$

$$+ \Sigma((\gamma k)\gamma^\beta - k^\beta) \bar{\Sigma}(\partial X^\mu + i(k\psi)\psi^\mu) (\bar{\partial} X_\beta - i(k\bar{\psi})\bar{\psi}_\beta) e^{ikX} \left. \right] \quad (22)$$

The computation of the relevant 3-point correlation function gives the following contribution to the  $\beta$ -function:

$$\begin{aligned} & \frac{1}{2}(z_1 - z_2)(z_2 - z_3)(z_3 - z_1)(\bar{z}_1 - \bar{z}_2)(\bar{z}_2 - \bar{z}_3)(\bar{z}_3 - \bar{z}_1) \times \\ & \times e_\mu(k_1)e_\nu(k_2) < V_D^\mu(k_1)V_D^\nu(k_2)V_\Phi > = \frac{3}{2}(k_1e(k_2))(k_2e(k_1)) + 2(e(k_1)e(k_2)). \end{aligned} \quad (23)$$

The contribution to the effective Lagrangian is therefore  $\sim (\frac{3}{2}(k_1D(k_2))(k_2D(k_1)) + 2D^2)\Phi$ . We see that the coupling (23) is reproduced by the following terms in the effective action:

$$\begin{aligned} S_{eff}^{DD\Phi} & \sim - \int d^{10}X \sqrt{g} \left( \frac{1}{4} e^{3/2\Phi} D_{\mu\nu} D^{\mu\nu}(X) + e^{2\Phi} D^2(X) \right) = \\ & = - \int d^{10}X \sqrt{g} \left( \frac{1}{4} e^{3/2\Phi} D_{\mu\nu} D^{\mu\nu}(X) + e^{2\Phi} \frac{\mu}{2} D^2(X) \right) \end{aligned} \quad (24)$$

Here  $D_{\mu\nu}$  is the massive gauge field strength;  $\mu = k^2$  is the mass of the gauge vector boson. Again, after the inverse rescaling of the metric tensor  $g_{\mu\nu} \rightarrow e^{-\frac{4\Phi}{D-2}} g_{\mu\nu}$  in order to bring the kinetic term to the standard form as in (2) we find that

$$S_{eff}^{DD\Phi} \sim - \int d^{10}X \sqrt{g} \left( \frac{1}{4} D_{\mu\nu} D^{\mu\nu}(X) + \frac{\mu}{2} D^2(X) \right), \quad (25)$$

i.e. we see that the massive vector boson with  $k^2 = 2$  is coupled with a dilaton in a non-standard way as in (3), as we expected. The straightforward proof of this proposal for other massive RR modes (tensors of higher ranks and with bigger masses) is principally the same though it requires much more cumbersome calculations. We hope to perform these computations in future papers, as well as the computations for the higher loops in the RR sector.

### Conclusion

We have shown that, in the lowest order of the string perturbation theory both massless and some massive Ramond-Ramond states have unusual coupling with a dilaton. This is related to the factor  $e^{\Phi(X)}$  in the worldsheet action of the dilaton that has been found in (20). Just like the term  $\sim R^{(2)}\Phi(X)$  in (1) appears as after the normal ordering of the vertex operator of a graviton, the worldsheet term (20) should also be "generated" by the normal ordering of a certain generally covariant vertex operator in the superstring theory which is yet unknown. To find the expression for this vertex operator would be very helpful in our understanding of the string perturbation theory in case of the supersymmetric background. Also, the conjecture about the relation of the dilatonic factors to the insertions of Ramond-Ramond states should be proved in details. It should therefore be necessary to consider higher order corrections in the Ramond-Ramond sector.



### **Acknowledgements**

I'm very thankful to D.Friedan,A.M.Polyakov and S.Shenker for many helpful comments.The work was supported from the Grant of the High Energy Theory group of Rutgers University.

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